

BUBBLE WALL PROFILES IN SUPERSYMMETRIC MODELS^aPeter John^b*Institut für Theoretische Physik, Philosophenweg 16,
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We present solutions to the equations of motion for bubble wall profiles in the minimal and a non minimal supersymmetric extension of the Standard model. We discuss the method of the numerical approach and present results for the two models (MSSM and NMSSM).

1 Introduction

For the emergence of a baryon asymmetry of the Universe the Sakharov conditions necessarily demand deviation from thermodynamical equilibrium. This condition is fulfilled in first order phase transitions. They take place via nucleation of bubbles separating the symmetric from the broken phase. A first order phase transition might occur at temperatures around the electroweak scale. It turned out that in the Standard Model (SM) there is no phase transition at all for Higgs masses larger than 72 GeV. Baryon number generation at the electroweak scale therefore requires more complicated models with additional light scalar fields such as the MSSM or NMSSM. In the MSSM there is a window for electroweak baryogenesis and an upper bound for the Higgs mass of about $m_H < 105$ GeV with a light stop¹ of mass $m_{\tilde{t}_R} < m_{top}$. In the NMSSM the bound on the Higgs mass is even weaker².

Having established the existence of a first order phase transition one can start the actual calculation of the baryon asymmetry itself. There are several mechanisms described in the literature. All of them need the knowledge of the profile of the bubble wall during the phase transition. The kink ansatz in many situations is a good approximation but it might be interesting to have a more refined description and to determine which deviations occur in the presence of potentials depending on two or more Higgs fields and eventually on CP violating phases³. Having the exact profile one can investigate the dynamics of expanding bubbles and calculate the baryon asymmetry.

To determine the bubble wall profile beyond a simple ansatz we have to solve the equations of motion numerically. In the case of more than one scalar

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field this is a highly nontrivial task since simple methods like overshooting-undershooting fail. So one has to use methods which, beginning with an ansatz, converge to the actual solution. They are sometimes called “relaxation methods”.

We first have to find the equations of motion. In field theory they can be derived via Euler Lagrange equations from the Lagrangian density which has the general form $\mathcal{L} = (D_\mu \Phi_i)^\dagger (D^\mu \Phi_i) + V(\Phi_i, T)$ for several Higgs fields Φ_i (plus, eventually, CP violating phases). Here D_μ is a covariant derivative and V denotes the effective potential. We show results of investigations of the MSSM and the NMSSM.

1.1 Critical Bubble

At the nucleation temperature the free energy becomes positive and the bubbles start to grow until they fill up the entire space. The profile of the radial symmetric bubbles along the radius r is determined by the equations of motion for the critical bubble (“bounce”):

$$0 = \frac{\partial^2 \phi_i}{\partial r^2} + \frac{2}{r} \frac{\partial \phi_i}{\partial r} - \frac{\partial V}{\partial \phi_i} \quad i = 1 \dots N \quad (1)$$

with boundary conditions $\frac{\partial \phi_i}{\partial r} = 0|_{r=\infty}$ and $\phi_i = 0|_{r=\infty}$, where $N = 2, 3$ for the MSSM and NMSSM respectively.

1.2 Stationary Bubble or Domain Walls

Constraining to a stationary wall with velocity v_w at a late time t where the wall is already almost flat we are left with only one spatial dimension $x = z - v_w t$ perpendicular to the wall and have the equations

$$0 = \frac{\partial^2 \phi_i}{\partial x^2} - \frac{\partial V}{\partial \phi_i} := E_i(x) \quad (2)$$

with two boundary conditions, e. g. $\frac{\partial \phi_i}{\partial x} = 0|_{x=\infty}$ and $\phi_i = 0|_{x=-\infty}$. We will discuss now the method and show solutions calculated with it.

2 Solutions

2.1 The Method

Since the overshooting-undershooting method fails we have to devise another method. We here minimize the functional of squared equations of motion.

Then, solving the eqs. of motion means finding field configurations for which the functional

$$\mathcal{F} = \int_{-\infty}^{+\infty} dx (E_1^2(x) + E_2^2(x) + \dots + E_N^2(x)) \quad (3)$$

is zero, which is achieved by minimizing \mathcal{F} . This method has already been used^{4,8} for the critical bubble of the MSSM.

Applying the minimization method we have to solve a boundary value problem. Thus we have to make an ansatz for every function for which we want to find the time development which fulfills the boundary conditions.

2.2 The Algorithm

The numerical algorithm works in two steps:

1. Find an ansatz as close as possible to the exact solution by low dimensional minimization of \mathcal{F} with special ansatz configurations.
2. High dimensional minimization of \mathcal{F} by discretizing every field function on a grid and minimization with respect to the function values.

In the MSSM the kink ansatz is quite appropriate to get a good starting configuration for the minimization procedure of step 1. In the NMSSM the improved kink ansatz is not that good but nevertheless still appropriate. With only a few parameters L_i and \hat{x}_i this is a low dimensional minimization procedure. In the MSSM we only have two (L_1, L_2), the offset \hat{x} is negligible, in the NMSSM five parameters ($L_{1/2/3}, \hat{x}_{1/2}, \hat{x} = 0$). We use as ansatz configurations:

$$\phi_i^{kink} = \frac{v_i}{2} \left(1 + \tanh\left(\frac{x}{L_i} + \hat{x}_i\right) \right), \quad i = 1 \dots N \quad (4)$$

With N fields on a grid with M space points the second step is a $N \times M$ dimensional minimization, which must be performed with fast converging methods. The results here are obtained by $N = 3$ fields, $M \sim 60 - 100$ grid points and Powell's quadratically converging method. The derivatives of the differential equations are discretized with three and four point formulae, the integrals of \mathcal{F} are performed with an extended Simpson rule. The field configurations are interpolated by splines^{5,6}. For more details on the algorithm, see also⁵. The worst problem doing numerics is the existence of spurious minima. Besides real solutions to the equations there are fake minima due to the numerical representation and solutions due to the fact, that $\delta E^2 = 0$ is fulfilled not only for $E = 0$ but also for $\delta E = 0$. One can perform checks to rate the minima found (see^{5,7}).

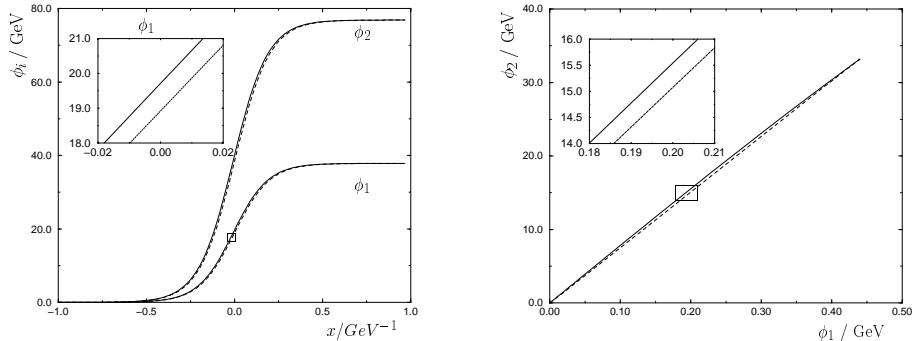


Figure 1: Solutions for the MSSM. Left: Solution (solid) configuration and *tanh*-ansatz (dashed). Only for unphysically small values of $m_A \approx \mathcal{O}(10 \text{ GeV})$ the deviation from the straight line (dashed) becomes appreciable. Wall widths are around $L \approx 10/T_c - 30/T_c \text{ GeV}^{-1}$.

2.3 Applications

We will show now applications of the described method. First, in figure 1 we present solutions of the bubble wall profile. The deviation from the straight line is small but nevertheless responsible for the actual amount of baryon asymmetry. Figure 2 shows the same for the three field case of the NMSSM. There we have a considerable deviation from the straight line and, additionally, a stronger deviation from the the extended kink ansatz. This demonstrates also the importance of a general solution method. Figure 2 shows also the path of the mechanical analogue of a rolling marble along the ridge of the potential.

These results indicate the general behaviour of solutions in theories with more than one Higgs field. Now questions of metastability of wrong minima can be investigated with higher accuracy and better reliability. This is one step for a more precise calculation of the actual amount of the baryon asymmetry of the universe.

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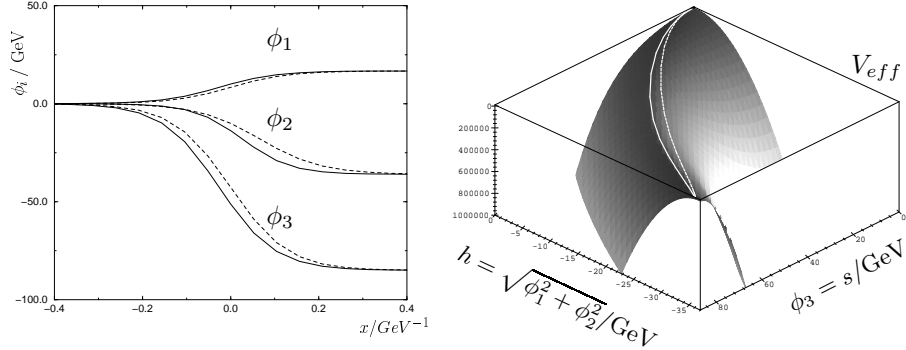


Figure 2: Left: Solutions (solid) for the NMSSM compared to \tanh -ansatz with offsets (dashed). The deviation from the ansatz is significant. Wall widths are around $L \approx 15/T_c$ GeV^{-1} . Right: Tunneling configuration on the negative potential; path of “marble” analogue. $\phi_3 = s$ is the singlet field of the NMSSM.

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